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Optimal Placement of Damped Struts Using Simulated Annealing

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Presented is the optimization of the placement of damped structural elements in a two-dimensional truss network. This work presents a modification of a heuristic optimization technique called simulated annealing, to improve the speed of convergence. Rather than setting the annealing temperature schedule a priori, it is modified according to the change in cost functional. For a single mode of vibration, a comparison is made between the simulated-annealing results and the true optimal locations, as determined by the modal strain locations. As expected, simulated annealing returns a near-optimal solution from a variety of initial configurations. Modal weighting factors are utilized to examine multiple modes, and again the modal strain locations are used for comparison.

Nomenclature

A = state matrix
E_d = dissipated energy, J
E_i = internal dissipative energy, J
E_{pj} = energy dissipated by the pth passive damper for the jth mode
I = number of possible locations
I^* = number of dissipative elements
k_b = Boltzman constant, J/K
Pr = Boltzman probability
T = temperature
V_j = modal strain energy, J
V_j^k = elemental modal strain energy, J
Z^j = normalization constant, dimensionless
z_0 = state vector of the structural system
β_j = modal weighting factor
ΔE = change in dissipated energy, J
ζ_i = modal damping ratio
η_j = modal loss factor
η^k = elemental loss factor
θ_k = decrement control parameter, J
ω_i = modal natural frequency, rad/s

Introduction

IN the control of large flexible space structures, the placement of active actuators and/or passively damped elements must be considered in the design of these systems. When deciding where to place passive dampers, locations are usually assigned on the basis of modal strain energy (MSE) principles. This method of placement would hold when considering only a single mode of vibration in a simple structure. However, when considering more than one mode in a complex structure, say a modal cluster, it is not so apparent what the optimal placement for the passive member is. Using a combinatorial optimization approach has been shown to have a high computation cost. It then becomes necessary to use a heuristic-based technique that renders near-optimal solutions at a low computation cost. A study was performed on a two-dimensional 20-bay truss, where motion was restricted to the horizontal plane. When a structure is spatially discrete and a finite number of passive members are to be placed in the structure, an optimal solution can be obtained by using a combinatorial optimization scheme. For the case presented in this study, 487,635 combinations would have to be calculated to obtain the optimal solution. To avoid such a large amount of computation, simulated annealing was used to sample this enormous space and find a near-optimal in a more feasible manner.

Typically, when considering a single mode of vibration, the placement of passive members in a structure is based on the amount of damping that the configuration of passive members contribute to the structural mode. The MSE relation implies that the optimal locations for member placement, for a single mode, would be the elements that contain the highest strain energies.[1] However, when the damping in a cluster of modes is to be optimized, it is not so apparent where the passive members should be placed. The modal strain energy considers only one mode, and no quantitative analysis can be made regarding the overall structural response. To determine the overall effect of a set of damped struts distributed in a truss, the physical response of the structure must be examined. Basing the cost functional on this response, one could then use an optimization algorithm to handle a large number of modes simultaneously.

For a single mode, a comparison the optimal configuration found is made with the high-modal-strain locations, to determine the proximity of the optimal solution to the true optimal, as determined by the MSE. For the multimode case, the MSEs give us only a qualitative measure of the solution found by simulated annealing.

Previous work done used the simulated-annealing method for the selection of active-member locations in adaptive structures.[2] This work considered three cases for the placement of active and passive members in a 54-member cantilevered boom. It was concluded that numerous trials using random initial configurations were needed to predict the best locations for the active members when considering a large number of possible locations.

This study examines the use of simulated annealing to optimally place four viscoelastic members in a two-dimensional (80-member) experimental test-bed facility. The structure, a two-dimensional planar truss, is an experimental test bed for structural control and vibration suppression. Only the 60 members with the highest modal strains are considered as candidate locations. Viscoelastically damped elements were constructed, and their damping was characterized. The passive members designed are similar in nature to those proposed in Ref. 3.

This truss was modeled using finite elements, and its damping characteristics, as well as those of the damped elements, were used in the optimization study. The optimization algorithm presented in

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this work differs from previous work in the way that the annealing temperature schedule is selected. The annealing temperature is a control parameter that allows the acceptance of nonimproving solutions. In previous work, the schedule was chosen a priori or chosen based on experience with a particular problem solution. The control parameter, the temperature schedule, is now based on the degree of improvement in the cost functional and has been automated.

Optimization Algorithm

The performance criterion was derived in Ref. 4 and is restated as follows for the case of selecting passive member locations in a two-dimensional truss structure:

$$E_d = E_i + \sum_{j=1}^{m} \beta_j E_{pj} \tag{1}$$

where E_i is evaluated as

$$E_i = z_0^T \int e^{A^T t} \begin{bmatrix} 0 & 0 \\ 0 & \text{diag}\{2\zeta_i\omega_i\} \end{bmatrix} e^{At}\, dt\, z_0 \tag{2}$$

E_{pj} is given by

$$E_{pj} = z_0^T \int e^{A^T t} \begin{bmatrix} 0 & 0 \\ 0 & \text{diag}\{\eta_j\omega_j\} \end{bmatrix} e^{At}\, dt\, z_0 \tag{3}$$

and β_j is the weighting factor for the passive-member contribution in the jth mode. The state matrix A is defined by

$$A = \begin{bmatrix} 0 & I \\ -\text{diag}\{\omega_i^2\} & -\text{diag}\{2\zeta_i\omega_i\} \end{bmatrix} \tag{4}$$

The modal loss factors can be computed using the following relation:

$$\eta_j = \sum_{k=1}^{n} \frac{\eta^k V_j^k}{V_j} \tag{5}$$

where η_j is the loss factor of the jth mode, η^k is the loss factor associated with the kth passive member, V_j is the strain energy of the jth mode at a given amplitude, and V_j^k is the strain energy in the kth element when the structure is deformed in the jth mode shape at the same amplitude. Whereas the actual stiffness of a passively damped structural element has a complex modulus, Eq. (5) implicitly assumes a viscous approximation of the damping. Often, for metallic structural elements, the loss factors associated with the truss elements are assumed negligible and only the loss factors associated with the passive members are considered. It is apparent that changing the configuration of the passive members in the truss structure changes the modal loss factors and directly affects the performance criterion.

When the available locations for placement of the passive members are spatially continuous, a gradient-based optimization method is sufficient. However, when these locations are spatially discrete, the optimization becomes a combinatorial problem. The computational complexity of a combinatorial problem can be expressed in the following form[3]:

$$n = \frac{I!}{I^*!(I - I^*)!} \tag{6}$$

Clearly, the number of possible combinations becomes rather large even for structures of small size. Therefore, instead of performing an exhaustive combinatorial optimization, a heuristic-based approach was used that renders near-optimal solutions in a less extensive search.

Iterative Improvement

When finding an approximate solution, the most common technique used is an iterative improvement method where only improvements in the performance criterion are accepted. The algorithm starts with an initial configuration of passive members, x_0, and then perturbs the member placements so a new configuration x is formed. A comparison between the respective performance criteria is made, and if the dissipation energy value for x is greater than the value for x_0, the configuration is updated or otherwise rejected. Thus, only improvements in the energy dissipation rate are accepted. This perturbation of member placement is repeated until no further improvements in the performance criterion can be achieved.

This type of optimization has a tendency to get trapped at local optima. Without any means of climbing away from these local values, a global value may not be found. Also, the solution is affected by the initial configuration chosen. Repeating the algorithm at various starting points does not guarantee that a global optimum can be found. Simulated annealing is a modified iterative improvement technique that provides a method that avoids becoming trapped at local optima, so that the solution is less dependant on the initial trial configuration.

Simulated Annealing

The simulated-annealing algorithm was developed in Ref. 5 and is based on the analogy between the annealing of solids and the problem of solving large combinatorial optimization problems. Annealing is a process in which a solid is heated and then slowly cooled, so that all particles will arrange themselves in a low-energy ground state. During the cooling process, the solid is allowed to reach thermal equilibrium at each temperature value T. The probability of being in a state with energy E is given by the Boltzmann distribution[2]

$$Pr = \frac{1}{Z(t)} e^{-E/k_B T} \tag{7}$$

where $Z(T)$ is a normalization factor depending on the temperature T. The factor $e^{-E/k_B T}$ is known as the Boltzmann factor. As the temperature decreases, the Boltzmann distribution concentrates on the states with the lowest energy. When the temperature approaches zero, only the minimum-energy states have a nonzero probability of occurrence. However, if the temperature is lowered rapidly, the solid does not reach thermal equilibrium for each temperature, which causes metastable amorphous structures rather than a low-energy lattice structure.

Metropolis Algorithm

To numerically simulate the behavior of particles in thermal equilibrium for a fixed value of temperature T, Metropolis et al.[6] proposed a Monte Carlo method that generates sequences of states of the solid. For example, given a current state of the solid, characterized by the position of the particles, a small randomly generated displacement of a particle is applied and the energy is evaluated as follows. If the difference in energy ΔE between the current state and the perturbed state is negative, i.e., if the displacement of a particle has resulted in a lower energy of the solid, then the process is continued with the new state. If $\Delta E \geq 0$, then the probability of acceptance of this perturbed state is governed by $e^{-E/k_B T}$. This rule of acceptance is referred to as the Metropolis criterion. After a large number of perturbations, using the acceptance criterion, the system eventually reaches thermal equilibrium for the fixed value of temperature.

Simulated-Annealing Algorithm

Using the Metropolis algorithm, which numerically simulates the behavior of particles in a solid at a finite temperature, Kirkpatrick[7] developed an equivalent method for use in a combinatorial problem. In this algorithm, the configurations assume the role of states of a solid, whereas the cost function E and the control parameter θ take the roles of energy and temperature, respectively. The simulated-annealing algorithm is viewed as a sequence of Metropolis algorithms evaluated at decreasing values of the control parameter. Starting at a high value for θ and a initial configuration i, another configuration j can be generated the same way as defined in the iterative improvement algorithm. Let $\Delta E_{ij} = E(j) - E(i)$; then the probability of configuration j being an acceptable perturbation is

given by 1 if $\Delta E_{ij} \geq 0$, and by $e^{-E/\theta}$ if $\Delta E_{ij} \leq 0$. The acceptance criterion is implemented by picking random numbers from a uniform distribution on [0, 1) and comparing with $e^{-E/\theta}$. Note that this is the Metropolis criterion modified for the problem where the cost function is to be maximized instead of minimized. Thus, there is a probability of continuing with a configuration which has a lower cost function than the current configuration. The control parameter θ is then lowered in steps, with the system being allowed to reach equilibrium at each step as described previously. The algorithm is terminated at some small value of θ, where the probability of acceptance is at a value for which virtually no decrease in the cost function is accepted. This final configuration is then considered the solution for the optimization problem. The simulated annealing algorithm is described in Fig. 1.

Three parameters essential for the implementation of the simulated annealing algorithm are as follows: 1) the initial value of the control parameter, θ_h, 2) the number of perturbations generated at each θ, and 3) and the decrement of the control parameter θ. These parameters affect the speed of the algorithm and the quality of the final configuration. A simple approach is to choose a value for θ_h that allows a large percentage (e.g., 80–90%) of nonimproving configurations to be accepted. The number of configurations generated at each θ is selected to allow equilibrium to take place before decreasing θ. For example, if there has been no improvement in the cost function after a number of perturbations, then equilibrium can be assumed to have taken place and θ can be decreased. The decrement of θ is chosen such that it allows only small changes in the value of θ. The equation used for decreasing θ is

$$\theta_{k+1} = \alpha\theta_k \tag{8}$$

where $\alpha = 0.9$ is a typical selection. The variation of this parameter also requires the user to define the number of iterations before an update to θ should occur. The update needs to be stated a priori and requires a good deal of user experience with the particular problem at hand. As an alternative to keeping θ constant throughout the inner loop, θ may be allowed to vary proportionally with the current optimal value of the cost function. The control parameter is modified in the inner loop when a configuration is accepted, i.e.,

$$\theta = \frac{\Delta E_{ij}}{\Delta E_{ij\,\text{last}}}\theta_{\text{last}} \tag{9}$$

where $\Delta E_{ij\,\text{last}}$ and θ_{last} are the cost and control parameter associated with the last accepted configuration. This modification allows θ to fluctuate substantially at the beginning of the algorithm when a high percentage of configurations are accepted. As a result, the number of computations necessary to attain equilibrium at high values of θ is greatly reduced. It also allows the update of θ to be determined automatically and not on a schedule chosen a priori.

Table 1 Theoretical natural frequencies of the truss

Mode	ω_n, Hz
1	1.64
2	9.74
3	25.54
4	30.25
5	45.53
6	68.70
7	88.85
8	92.61
9	117.35
10	139.96

```
i = initial configuration
E(i) = cost(i)
θh = initial value for control parameter
|**outer loop**|
while θ > θ0 (stop if criterion is not satisfied)
    |**inner loop**|
    while (equilibrium is not satisfied)
        perturb (configuration i → configuration j , ΔEij=E(j)-E(i))
        if ΔEij > 0 then accept
        else
            accept only with probability
            e^(-E/θ) > random [0,1) then accept
        end
        if accept then update configuration E(i)=E(j)
    end
    decrement control parameter θ = αθ
end
```

Fig. 1 Description of the annealing algorithm as coded in MATLAB®.

Structural Parameter Determination

This study was conducted on a test-bed facility at the U.S. Air Force Academy.[8] The test bed consisted of a 20-bay planar truss structure supported by ball bearings to restrict motion to the horizontal plane. Steel bars were mounted across every bay to add weight and to support the truss on the ball bearings. A schematic of the truss is shown in Fig. 2. This schematic shows the members of the truss structure numbered with the assigned element values from a finite-element analysis.

A finite-element model was evaluated using a commercial finite-element package, I-DEAS.[9] A modal analysis was performed to obtain the natural frequencies and modal strain energies for the first 10 modes. Table 1 gives the natural frequencies for each mode.

Figure 2 shows the 80-member truss. When examining the first 10 modes, the horizontal members (the members parallel to the base) do not have any strain associated with them and are not considered as locations for passive-member placement. The strain energy in a member is a function of the mode shape. Therefore, the percentage of total strain a member has reflects the type of mode shape that is being evaluated. Table 2 lists the first three modes and the percentage of total strain the members have in each mode. The mode shapes are given in the Appendix.

Figure 3 shows an experimental frequency response function (FRF) of the truss for the first four modes. The FRF also shows a good correlation between the experimental and theoretical natural frequencies. The response function was obtained by placing accelerometers at the tip of the truss.

The only experimental data that were used in the optimization algorithm were the loss factors of the passive members and the inherent damping in the structure. Assuming the finite number of passive members had the same characteristics, the loss factors associated with a passive damper were obtained by placing the passive damper in the truss and examining the damping ratios. Using the equation $\eta^i = 2\zeta_i$, which is valid at resonance, the loss factors were calculated and are listed in Table 3. The η^k values listed were used in the algorithm when evaluating the modal loss factors of the truss when the four passive members were placed randomly in the truss. That is, the inherent internal damping of the existing structure was not neglected in the optimization.

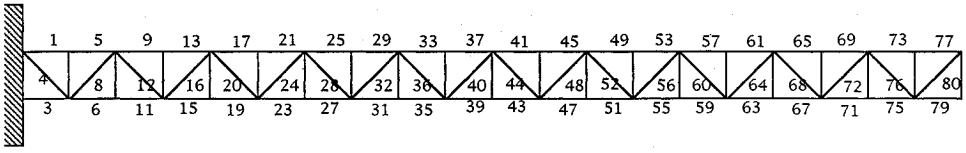

Fig. 2 Twenty-bay planar truss with element location numbers.

Table 2 Theoretical percentage of strain energy in each member for the first three modes

	Fraction of strain, %		
Element	Mode 1	Mode 2	Mode 3
1	8.30	4.74	2.00
3	9.54	8.26	6.93
4	0.12	1.36	3.26
5	8.30	4.74	1.99
6	7.15	2.23	0.28
8	0.12	1.34	2.97
9	6.08	0.67	0.27
11	7.15	2.23	0.27
12	0.12	1.27	2.32
13	6.08	0.67	0.28
15	5.11	0.03	1.22
16	0.12	1.15	1.43
17	4.23	0.17	2.88
19	5.11	0.03	1.23
20	0.12	0.98	0.59
21	4.23	0.17	2.87
23	3.44	0.88	3.01
24	0.12	0.78	0.08
25	2.74	1.92	2.95
27	3.44	0.88	3.01
28	0.11	0.56	0.05
29	2.74	1.92	2.93
31	2.14	3.01	1.43
32	0.11	0.35	0.46
33	1.62	3.95	0.48
35	2.14	3.01	1.42
36	0.10	0.18	1.03
37	1.62	3.95	0.46
39	1.19	4.53	0.02
40	0.09	0.06	1.45
41	0.84	4.62	0.61
43	1.19	4.52	0.02
44	0.08	0.00	1.49
45	0.84	4.61	0.62
47	0.57	4.27	2.28
48	0.07	0.02	1.14
49	0.37	3.59	3.59
51	0.57	4.26	2.28
52	0.06	0.08	0.59
53	0.37	3.58	3.61
55	0.22	2.71	4.81
56	0.05	0.17	0.14
57	0.12	1.81	4.39
59	0.22	2.70	4.79
60	0.04	0.25	0.00
61	0.12	1.80	4.39
63	0.06	1.03	3.52
64	0.03	0.30	0.21
65	0.02	0.47	1.89
67	0.06	1.02	3.47
68	0.02	0.30	0.56
69	0.02	0.46	1.88
71	0.01	0.15	0.76
72	0.01	0.25	0.79
73	0.00	0.02	0.12
75	0.01	0.14	0.73
76	0.01	0.15	0.68
77	0.00	0.02	0.11
79	0.00	0.00	0.00
80	0.00	0.00	0.29

The variation of these loss factors is indicative of the limitations of the MSE approximation. The MSE method assumes that the eigenvectors, or mode shapes, of the damped system are the same as those for an undamped system. This assumption works well for highly redundant structural systems where the loss of stiffness of a single truss element does not greatly effect the eigenvectors of the overall structure. However, when an element's stiffness is reduced by placement of a passively damped member, this can greatly affect the modal characteristics of the structure and hence compromise the MSE assumption. An averaged loss factor for the truss was assumed to be $\eta = 0.3242$. It was assumed that this loss factor was constant

Table 3 Experimental loss factors for a passive member

Mode	η
1	0.4546
2	0.3394
3	0.1788

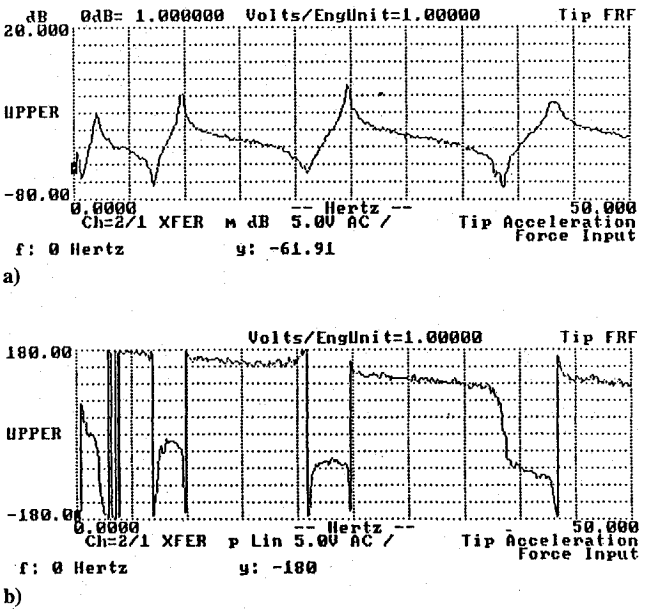

Fig. 3 Inertance transfer function for the 20-bay planar truss structure tip acceleration vs tip impulse: a) magnitude of acceleration/force (volts/volts) and b) phase (degrees).

for the various modes. We also assumed that the damped strut has the same stiffness as the one it replaces. This simple study on elemental loss factors for the damped elements illustrates the limitation of the MSE assumption in a truss without high redundancy.

Single-Mode Evaluation

The effectiveness of simulated annealing as an optimization tool for placement of passive dampers in a structure was examined first. Optimization trials were performed on only one mode of vibration at a time, because the results could then be easily compared with the MSE results.

For the first mode of vibration, the MSE method would place four passive dampers at member locations 1–4, which are shown in Fig. 4a. The locations are ranked in order of percentage of strain energy, the highest being marked with a circled 1. Since the final solution of a simulated-annealing trial is dependent on the initial configuration, 10 trials were performed from random initial configurations. Figure 4b displays the locations that appear in the final solution of each trial, denoted by hash marks. Comparing Figs. 4a and 4b shows simulated annealing was able to locate the members that contain the highest percentage of strain energy; however, a number of suboptimal locations were also selected. Figure 4b was constructed with the results of 10 trials. This qualitatively illustrates the limitation of the heuristic technique; the simulated annealing finds a nearly optimal solution. It shows that the algorithm is sampling its search space to find the best configuration possible in a limited number of iterations, rather than evaluating the whole search space. This comparison was also performed for modes 2 (15 trials) and 3 (10 trials), with the results given in Figs. 5 and 6, respectively.

The damping ratios achieved by the optimization characterized in Figs. 4–6 are summarized in Table 4. The optimal values of damping can be readily determined from the MSE, using the highest four strain locations for each mode. The maximum damping ratios are 0.1079, 0.0725, and 0.0678 for modes 1, 2, and 3, respectively. Although the annealing technique did often place some dissipative

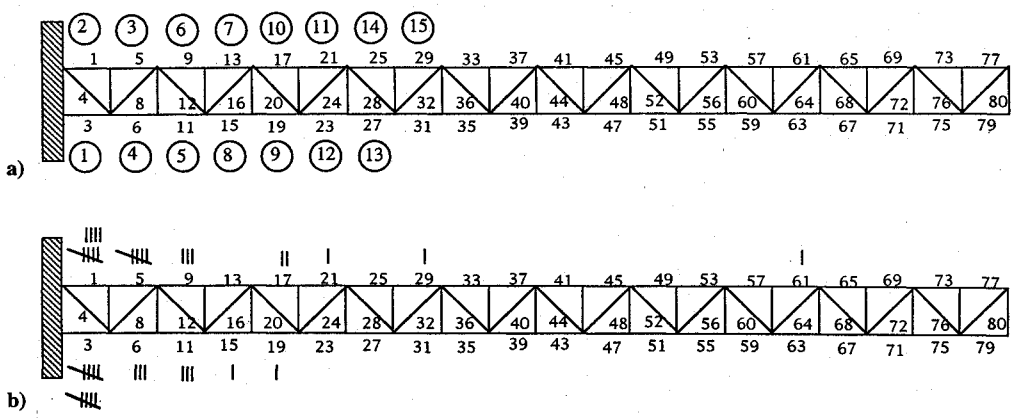

Fig. 4 Mode 1: Comparison between MSE and simulated annealing; a) high-strain-energy locations and b) locations selected by simulated annealing.

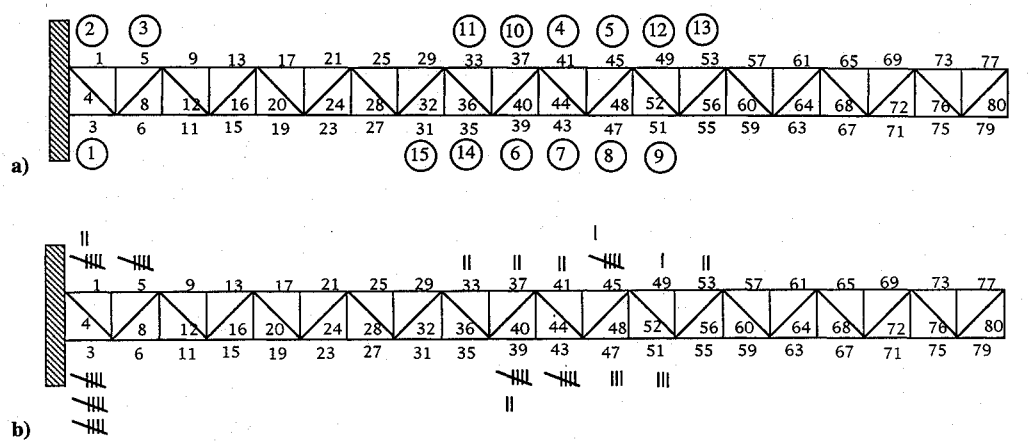

Fig. 5 Mode 2: Comparison between MSE and simulated annealing; a) high-strain-energy locations and b) locations selected by simulated annealing.

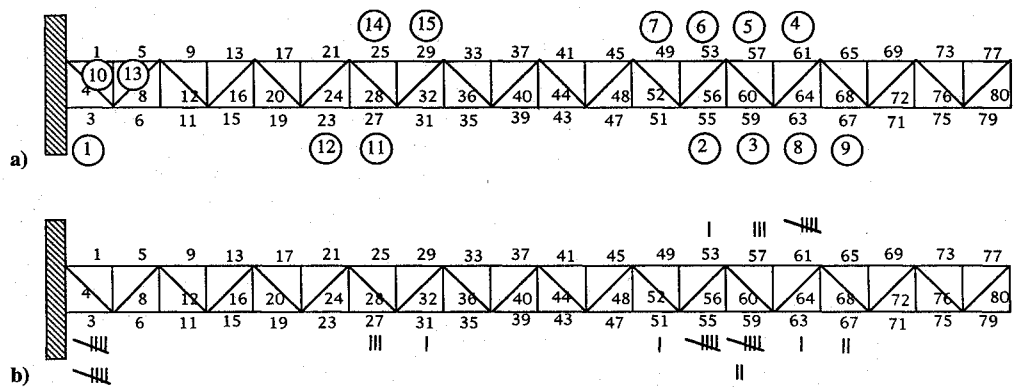

Fig. 6 Mode 3: Comparison between MSE and simulated annealing; a) high-strain-energy locations and b) locations selected by simulated annealing.

elements at suboptimal locations, it often found a solution that was physically near the true optimal locations. The routine relies, in part, on probability. Thus, by making several trial runs, a set of locations can be determined to perform a feasible combinatorial search.

Figure 7 shows an iteration history of the simulated annealing algorithm from a random initial configuration. The effect of the probability function is clearly visible. The algorithm initially accepts many nonimprovements in the cost function. Towards the end of the trial algorithm accepts fewer nonimprovements until finally it behaves much like an iterative improvement program, where only improvements are accepted. Simulated annealing has an advantage over iterative improvement in that it allows a means of climbing away from a local optimum. Starting from the same initial configuration, Fig. 8 shows the simulated annealing algorithm obtaining a solution that allows a higher energy dissipation than the iterative improvement method.

The convergence of the algorithm to a near-optimal solution is dependent on the annealing schedule. Previous work[4] that used simulated annealing for passive-member placement defined the annealing schedule as $\theta \rightarrow \alpha\theta$. A modification to this annealing schedule was recommended (see Eq. 6) that allowed the control parameter θ to vary within the inner loop, i.e., as a function of dissipated energy. This modification allowed the final solutions to converge to a solution and is shown in Fig. 9a. The graph shows six trials that start at random configurations and converge to a solution that does not depend as much on the initial configuration. Using these same initial configurations, Fig. 9b is a graph of the trials when the an-

Table 4 Damping ratios for single-mode optimization results

	Damping ratio		
Trial	Mode 1	Mode 2	Mode 3
1	0.1045	0.0714	0.0590
2	0.0947	0.0680	0.0620
3	0.1002	0.0718	0.0621
4	0.1079	0.0666	0.0569
5	0.0851	0.0715	0.0665
6	0.0899	0.0723	0.0640
7	0.0973	0.0725	0.0623
8	0.0942	0.0692	0.0678
9	0.0913	0.0725	0.0633
10	0.1013	0.0683	0.0635
11		0.0711	
12		0.0706	
13		0.0680	
14		0.0700	
15		0.0672	

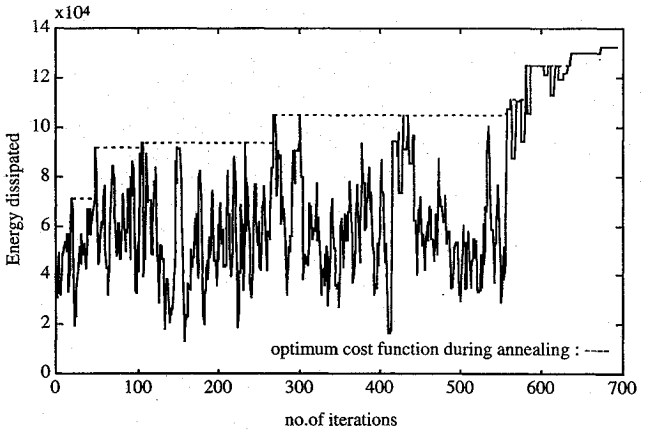

Fig. 7 Iteration of simulated annealing algorithm.

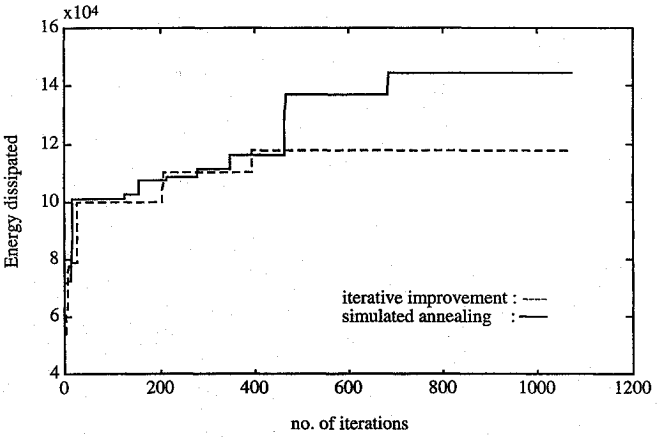

Fig. 8 Comparison between simulated annealing and iterative improvement.

nealing schedule was set as $\theta \rightarrow \alpha\theta$. This graph clearly shows the trials not converging to a single solution and that these trials are more dependent on the initial configuration. The effects of this modification demonstrate the need to examine the annealing schedule before any optimization can be performed. Figure 9 shows that the modification of the annealing schedule based on the cost functional yields faster convergence: 100–300 iterations for five of six trials in Fig. 9a as compared to 200–600 iterations for five of six trials in Fig. 9b.

Multiple-Mode Evaluation

In this section the results from the evaluation of multiple modes are presented. Combinations of modes 1 and 3 and modes 2 and

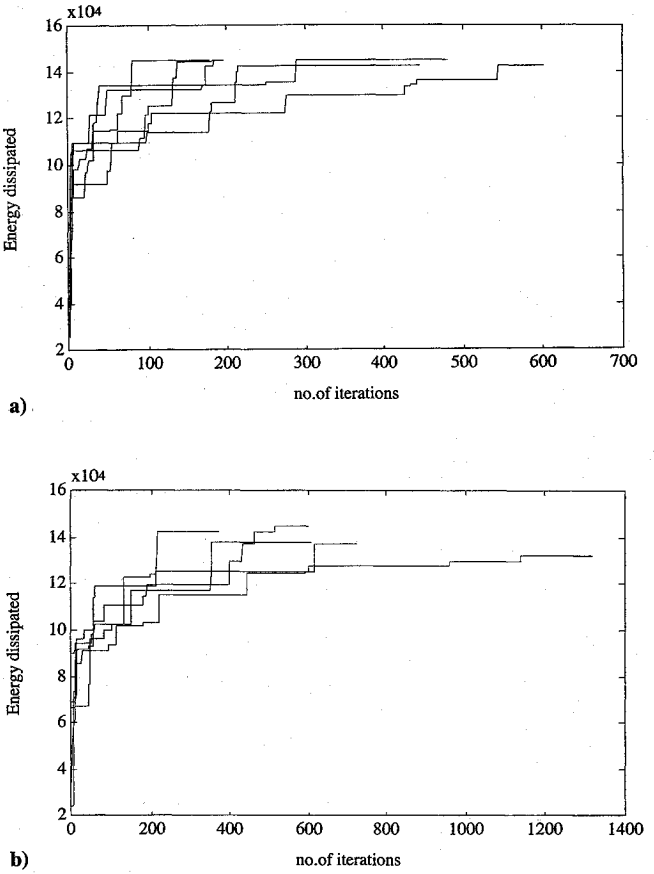

Fig. 9 Comparison of near-optimal solutions: a) modified simulated annealing schedule and b) original annealing schedule.

3 are evaluated simultaneously as clusters for the frequency bandwidth between 0 and 26 Hz. If a designer wants to damp more than one mode of vibration, then the MSE method cannot quantitatively predict the optimal location for a passively damped element. However, the MSE does provide insight, on a per-mode basis, and a qualitative measure of whether the heuristic optimization technique has chosen "good" candidate locations. This insight will be used to evaluate qualitatively the performance of the optimization of multiple modes for various weighting factors. Quantitatively, the performance of this multiple-mode optimization is examined by monitoring the variation in modal damping ratios for modes under consideration during a trial.

The weighing factors β_i for each mode i are set to a value that weights the contribution that each mode has in the optimization scheme. For example, if $\beta_2 = \beta_3 = 1$, modes 2 and 3 are considered as equal contributors in the optimization. If β_i for a mode is set to zero, then the mode is not considered in the optimization. If a mode is considered more strongly than another, say $\beta_2 > \beta_3$, then the energy dissipated in mode 2 is valued more than energy dissipated in mode 3.

Consider modes 1 and 3 as the cluster of modes in a frequency bandwidth which is to be controlled. When $\beta_1 = \beta_3 = 1$, the modes are considered to be equal contributors, and thus the placement of passive dampers in the truss should be at locations that are compatible with both modes. Figure 10 shows the results from 15 trials that started at random configurations. Four passive dampers were to be optimally placed in the truss to control modes 1 and 3. As can be seen from the figure, the optimization algorithm obtains solutions that would place these dampers at the constrained end of the truss. These locations are more suitable for mode 1 than for mode 3. If β_1 is decreased to 0.5 while $\beta_3 = 1$, only a few solutions would place dampers in locations suitable to mode 3, as shown in Fig. 11. These results can be explained by examining the mode shapes of modes 1 and 3 (see Appendix). In mode 1, the members that have the highest concentration of strain are located at the base of the truss, whereas in mode 3 only a few members at the base have high strain.

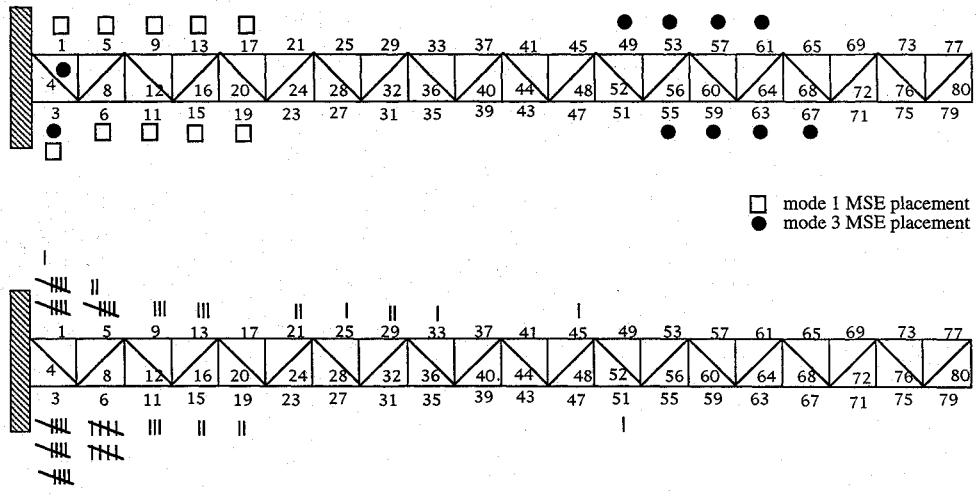

Fig. 10 Optimal placement of passive members for modes 1 and 3 where $\beta_1 = 1$ and $\beta_3 = 1$.

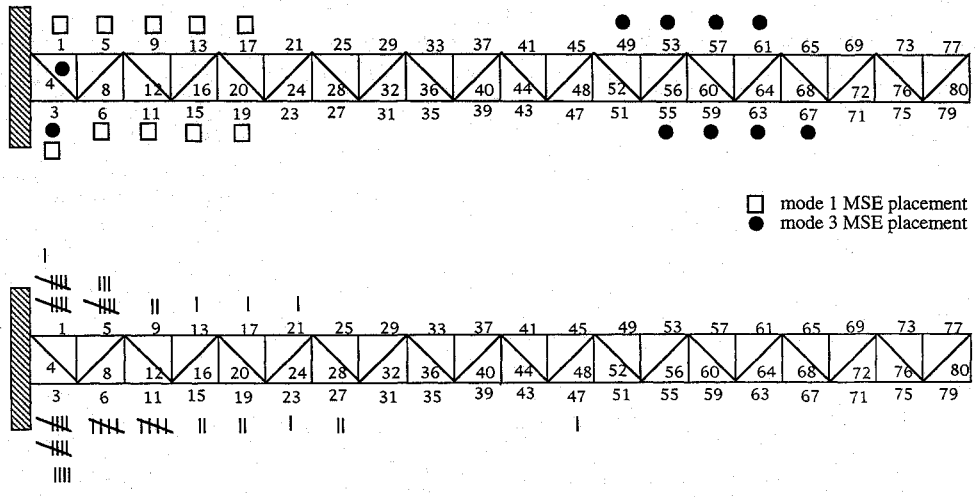

Fig. 11 Optimal placement of passive members for modes 1 and 3 where $\beta_1 = 0.5$ and $\beta_3 = 1$.

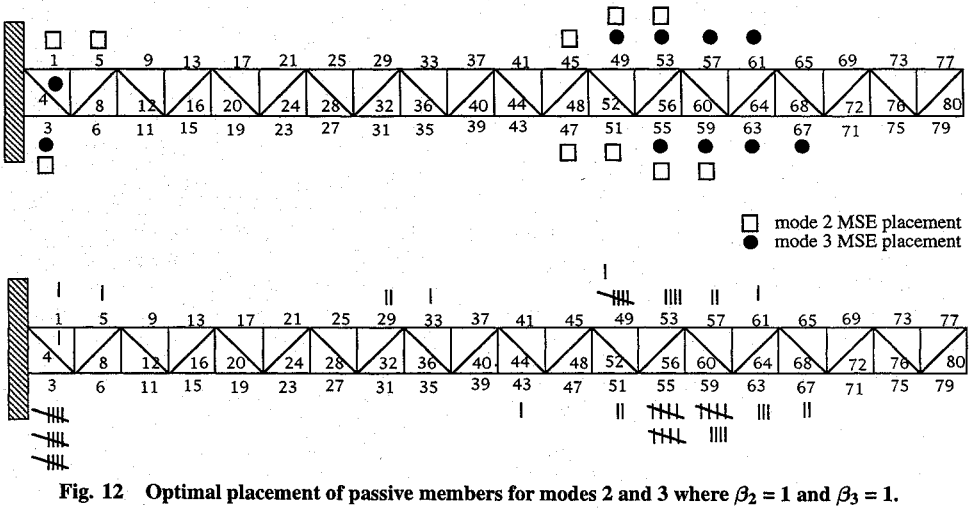

Fig. 12 Optimal placement of passive members for modes 2 and 3 where $\beta_2 = 1$ and $\beta_3 = 1$.

Members located in the midsection of the truss also contain a higher concentration of strain than members at the base for mode 3. These results suggest that mode shapes that contain a high concentration of strain in the same region would be better modes to consider in the algorithm.

With the previous conclusion in mind, modes 2 and 3 are now considered as a cluster of modes in the frequency range of 10–25 Hz. Referring to their respective mode shapes in the Appendix, it is noted that the highest concentration of strain in both modes lies in the midsection of the truss. Setting $\beta_2 = \beta_3 = 1$, the simulated annealing algorithm is performed for 15 trials, and the solutions are shown in Fig. 12. The results show the optimal locations distributed to high-MSE locations for mode 3 and mode 2. Now, changing the weighting factors to $\beta_3 = 0.5$ and $\beta_2 = 1$ gives solutions that are suitable to

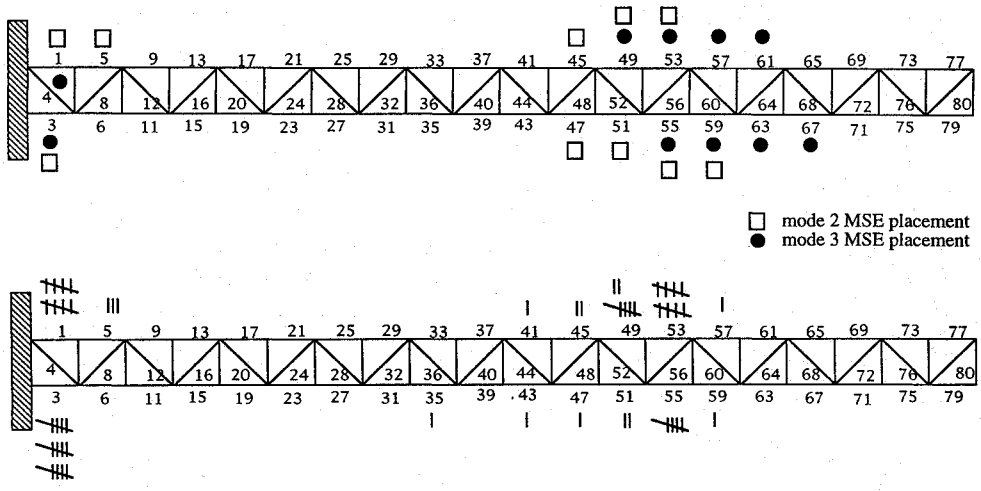

Fig. 13 Optimal placement of passive members for modes 2 and 3 where $\beta_2 = 1$ and $\beta_3 = 0.5$.

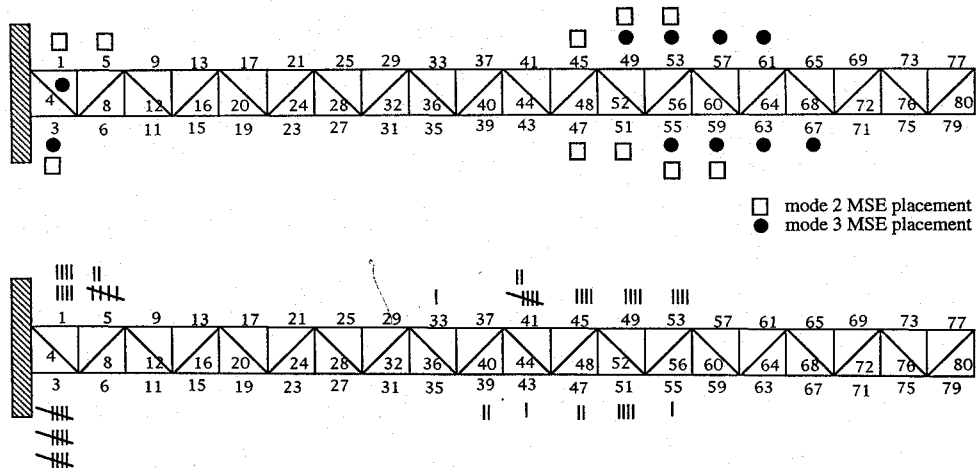

Fig. 14 Optimal placement of passive members for modes 2 and 3 where $\beta_2 = 1$ and $\beta_3 = 0.5$.

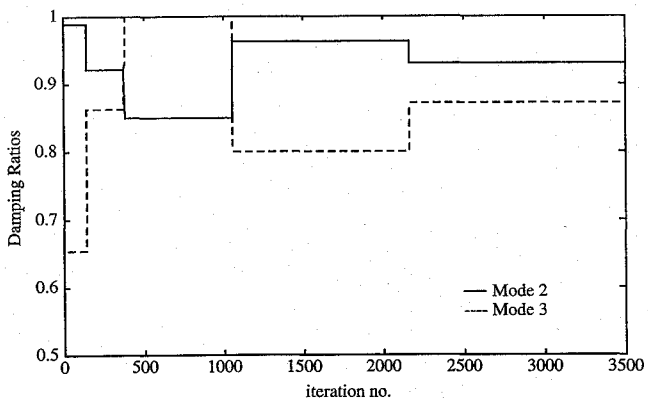

Fig. 15 Iteration history of damping ratios for mode 2 and 3.

both modes (see Fig. 13). By further reducing β_3, the algorithm tailors the solutions to locations that are more favorable to mode 2 (see Fig. 14). Therefore, the choice of the weighting factors are design-dependent and must be considered as important parameters for the optimization scheme.

As the algorithm generates random configurations to evaluate the cost function, the program retains the best solution to date. The solution is updated whenever improvements in the cost function are found. The configurations that yield improving solutions have modal damping ratios associated with them, and an evaluation of how the algorithm affects these damping ratios is now examined. Referring to Fig. 15, which shows the normalized modal damping ratios for modes 2 and 3 from the run that generated Fig. 12, a qualitative analysis of the mode can be performed. As the damping ratio improves for one mode, the damping ratio of the other mode is decreased until a solution is found that satisfies the cost functional. It is interesting to note that the number of iterations also increases as the search space becomes more complex.

Conclusions

The simulated annealing optimization scheme was examined. It was used to optimally select the positions of passive members in a 20-bay planar truss to increase the damping in the structure. A comparison with iterative improvement demonstrated how iterative improvement alone, without an annealing process, gets trapped in local minima. The control parameter for the temperature scheduling, which governs the probability of accepting nonimproving solutions, was written as a function of the cost. This modification was shown to improve the convergence of simulated annealing solutions in an example. When the temperature schedule is not based on the cost functional, then it must be selected a priori, i.e., the designer must "get a feel" for the optimization to determine the schedule. A comparison between the MSE relation and simulated annealing was performed to examine the performance of the algorithm both quantitatively and qualitatively. Simulated annealing was shown to obtain solutions that contain high-MSE locations for both a single-mode and a multiple-mode case. A qualitative analysis of multiple-mode optimization was performed, and the effects of weighting factors were investigated. Although simulated annealing selects suboptimal locations, it did yield solutions very close to optimal at a fraction of the numerical cost of a fully combinatorial search.

Appendix

Figure A1 shows the displacement for the first three modes of vibration of the 20-bay planar truss.

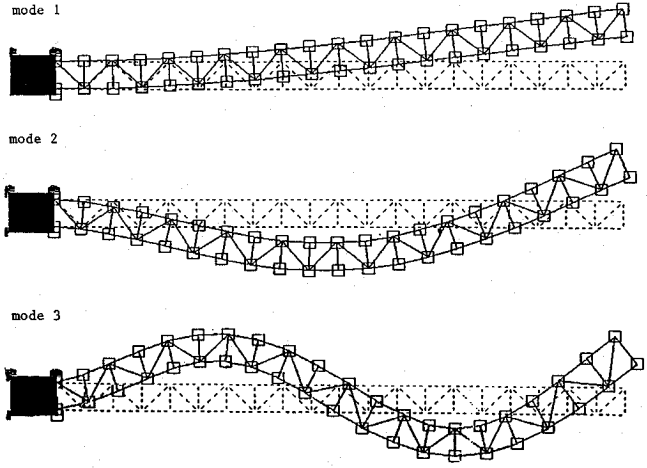

Fig. A1 First three modes of vibration.

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